Announcements

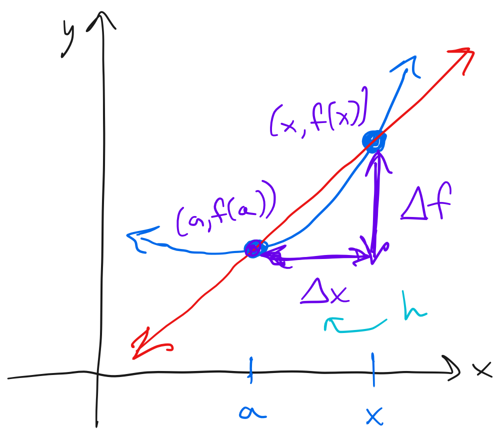
- No class or OH Monday (MLK Day)
- In-person OH Wed (Jan 19) 1-2pm at Art of Espresso
- Extra Credit problems for §2.6, §2.8 deadline extended to Mon 11:59pm

Today §3.1: Definition of the Derivative

Today in history: invention of the clarinet (1690)

Remember from §2.1: for a function f,

Secant line

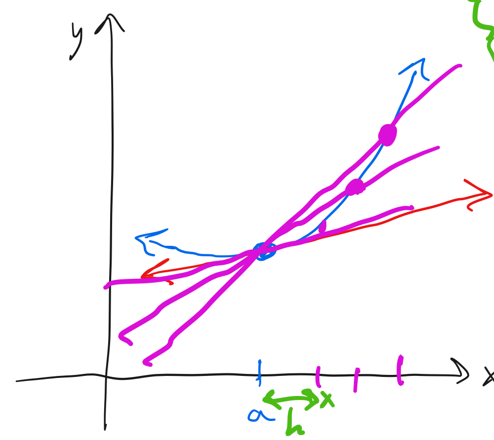


Easy to compute slope of secant line

$$\text{slope} = \frac{\Delta f}{\Delta x} = \underbrace{\frac{f(x)-f(a)}{x-a}}$$

Vocab called a "difference quotient"

Tangent line



To find slope of tangent line, look at secant lines with Δx smaller and smaller:

$$\text{slope} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

notice: can't substitute $\Delta x = 0$

"f prime of a"

Def The derivative of f at $x=a$, written $f'(a)$, is defined by

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h},$$

provided the limit exists.

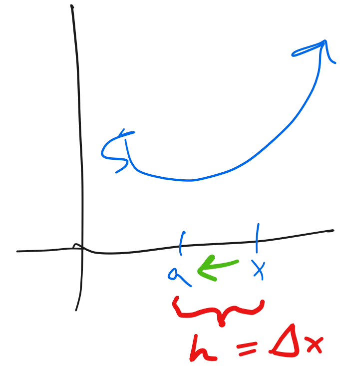
When the limit exists, we say that f is differentiable at $x=a$.

Note $\displaystyle\lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

by change of variables:

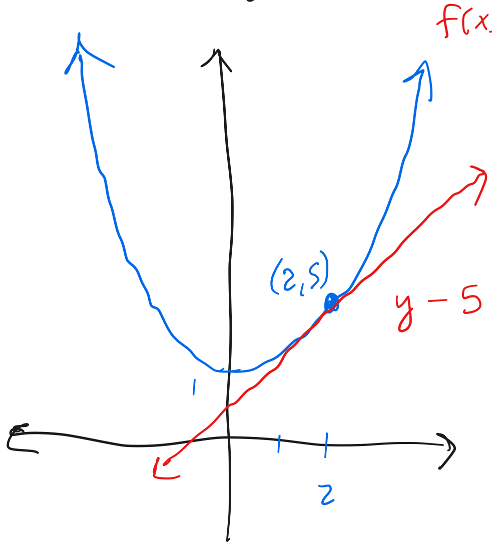
$h = x - a \iff x = a + h$

$h \to 0 \iff x \to a$



Ex Find an equation for the tangent line to the graph of $y = x^2 + 1$ at $x = 2$

$f(x)$



at $x = 2$,
$y = 2^2 + 1 = 5$

$y - 5 = m(x-2)$ point-slope form of the tangent line

Now all we need is the slope m of the line.

$$m = f'(2) = \lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h \to 0} \frac{(2+h)^2 + 1 - 5}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4} + 4h + h^2 + \cancel{1} - \cancel{5}}{h} = \lim_{h \to 0} \frac{\cancel{h}(4+h)}{\cancel{h}}$$

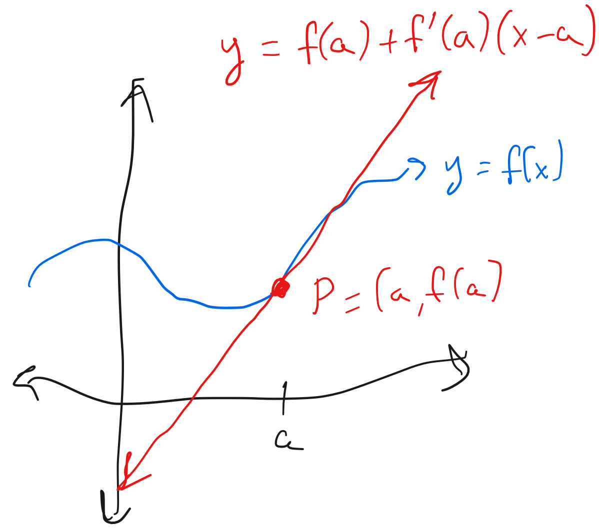
$$= 4 + (0) = 4$$

Thus, the tangent line is $y - 5 = 4(x-2)$

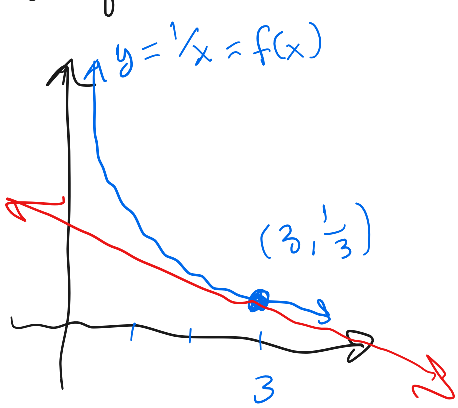
standard form $y = \underbrace{5}_{f(2)=f(a)} + 4(x - \underbrace{2}_{a=2})$

tangent line for $f(x)$: $y = f(a) + f'(a)(x-a)$

Def (Tangent Line) Suppose f is differentiable at $x=a$. The tangent line to the graph $y=f(x)$ at $P=(a,f(a))$ is the line through P with slope $f'(a)$.



Ex Find the slope of the tangent line to the graph $y=\frac{1}{x}$ at $x=3$.



$$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{3}{3(3+h)} - \frac{3+h}{3(3+h)}}{h}$$

$$\frac{1}{h}\left[\frac{3}{3(3+h)} - \frac{3+h}{3(3+h)}\right]$$

$$= \frac{1}{h}\left[\frac{3-(3+h)}{3(3+h)}\right]$$

$$= \frac{3-(3+h)}{h\cdot 3(3+h)}$$

$$= \lim_{h\to 0} \frac{\cancel{3}-(\cancel{3}+h)}{3(3+h)h} \leftarrow \cancel{3}-\cancel{3}-h$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{3(3+h)\cancel{h}}$$

$$= \lim_{h\to 0} \frac{-1}{3(3+h)} = \boxed{\frac{-1}{9}}$$

$h \to 0 \qquad 3(3+h)$

$= \dfrac{-1}{3(3+0)}$

<u>Ex</u> $f(x) = |x|$.

Find the following

$f'(2)$, and $f'(-2)$, and $f'(0)$